CHAPTER 15

Probability and Statistics

1. INTRODUCTION

The theory of probability has many applications in the physical sciences. It is of basic importance in quantum mechanics where results may be expressed in terms of probabilities (see Chapter 13, Schrödinger equation). It is needed whenever we are dealing with large numbers of particles or variables where it is impossible or impractical to have complete information about each one, such as in kinetic theory and statistical mechanics and a great variety of engineering problems. Statistics is the part of probability theory which deals with the interpretation of sets of data. You need statistical terms and methods every time you make a set of laboratory measurements. In this chapter, we shall discuss some of the basic ideas of probability and statistics which are most useful in applications.

The word "probably" is frequently used in everyday life. We say "The test will probably be hard," "It will probably snow today," "We will probably win this game," and so on. Such statements always imply a state of partial ignorance about the outcome of some event; we do not say "probably" about something whose outcome we know. The theory of probability tries to express more precisely just what our state of ignorance is. We say that the probability of getting a head in one toss of a coin is $\frac{1}{2}$, and similarly for a tail. We mean by this that there are two possible outcomes of the experiment (if we do not consider the possibility of the coin's standing on edge) and that we have no reason to expect one outcome more than the other; therefore we assign equal probabilities to the two possible outcomes. (See end of Section 2 for further discussion of this.)

Consider the following problem. You and I each toss a coin and look at our own coins but not each other's. The question is "What is the probability that both coins show heads?" Suppose you see that your coin shows tails; you say that the probability that both coins are heads is zero because you *know* that yours is tails. On the other hand, suppose I see that my coin is heads; then I say that the probability of both heads is $\frac{1}{2}$ because I don't know whether your coin shows heads or tails. Now suppose neither of us looks at either coin, but a third person looks at both coins and gives us the information that at least one is heads. Without this